SPE- 102430

History Matching Using the Ensemble Kalman Filter on a North Sea Field Case

Vibeke Haugen, SPE, Statoil ASA, Lars-Jørgen Natvik, Statoil ASA, Geir Evensen, Hydro, Aina Berg, IRIS, Kristin Flornes, IRIS, Geir Nævdal, SPE, IRIS

This paper was prepared for presentation at the 2006 SPE Annual Technical Conference and Exhibition held in San Antonio, Texas, U.S.A., 24–27 September 2006.

Abstract

Recently, the ensemble Kalman filter (EnKF) has been examined in several synthetic cases as an alternative to traditional history matching methods. Results from these studies indicate that the method can be useful for estimation of permeability and porosity fields.

Contrary to other history matching methods, the EnKF provides an ensemble of model realizations containing information of the uncertainty in the estimates. Moreover, the data is processed sequentially, which makes it possible to always have an updated model conditioned on the most recent production data. The method therefore seems promising for real time reservoir management. This paper presents a successful study for a North Sea field case, where real production data have been assimilated using EnKF.

Introduction

The ensemble Kalman filter (EnKF) developed by Evensen (1994, 2003) is a statistical method suitable for history matching a reservoir simulation model. It is a Monte Carlo method where errors are represented by an ensemble of realizations. The prediction of the estimate and uncertainty is done by ensemble integration using the reservoir model. Thus, the method provides error estimates at any time based on information from the ensemble. At data times, a variance minimizing scheme is used to update the realizations based on the production data. The EnKF provides a general and model independent formulation, and can be used to improve the estimates of both parameters and variables in the model. The method has previously been applied in a number of applications, e.g., with dynamical ocean models (Haugen and Evensen, 2002), in model systems describing the ocean ecosystems (Natvik and Evensen 2003a, b), as well as in applications within meteorology (Houtekamer et al, 2005). This shows that the EnKF is capable of handling different types of complex and non-linear model systems.

The method was first introduced into the petroleum industry in studies related to well flow modelling (Lorentzen et al 2001, 2003). Nævdal et al. (2002) used the EnKF in a reservoir application to estimate model permeability focusing on a near-well reservoir model. They showed that there could be a great benefit using the EnKF to improve the model through parameter estimation, and that this could lead to improved predictions. Nævdal et al. (2003) showed promising results estimating the permeability as a continuous field variable in a 2-D field like example. Gu and Oliver (2004) examined the EnKF for combined parameter and state estimation in a standardized reservoir test case, the PUNQ-S3 model. Gao and Reynolds (2005) compared the EnKF with the randomized maximum likelihood method and pointed out several similarities between the methods. Liu and Oliver (2005a, 2005b) examined the EnKF for facies estimation in a reservoir simulation model. This is a highly nonlinear problem where the probability density function for the petrophysical properties becomes multi-modal, and it is not clear how the EnKF can handle this. A method was proposed where the facies distribution for each ensemble member is represented by two normal distributed Gaussian fields, using a method named truncated pluri-Gaussian simulation (Lantuéjoul, 2002). Wen and Chen (2005) provided another discussion on the EnKF for estimation of the permeability field in a two dimensional reservoir simulation model, and examined the impact of the ensemble size. Lorentzen et al. (2005) focused on the sensitivity of the results with respect to the choice of initial ensemble using the PUNQ-S3. Skjervheim et al. (2005) used the EnKF to assimilate seismic 4D data. It was shown that the EnKF can handle these large data sets and that a positive impact could be found despite the high noise level in the data.

This paper considers the use of the EnKF for estimating dynamic and static parameters, focusing on permeability and porosity, in a field model of a Statoil operated field in the North Sea. The largest uncertainty in the model is expected to be related to the permeability values, especially in the upper part of the reservoir where it may approach 30%.

Simulation model

The simulation model is an eclipse 100 black oil model of the field located in the North Sea. The horizontal grid is 45 x 75 giving a resolution of 40 x 50 m (see Figure 1). There are 26 layers with thickness varying between 5 and 50 meters, with increasing resolution downwards, and a total of 45000 active cells. The simulation grid is the same as the geological model, thus no up-scaling has been used.

There are four oil producers and 2 gas injectors in the model. The drainage strategy is to produce the oil by injecting gas.

To obtain the original history match of the production and pressure data, tuning of vertical communication and the absolute permeability in the upper zones and pseudorizing of relative permeability in water was performed.

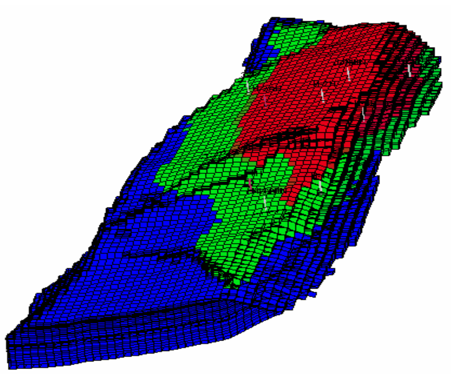

Figure 1. Simulation grid of the Statoil operated field in the North Sea.

Ensemble Kalman filter theory and setup

The optimal sequential approach to solve linear inverse problems was originally developed by Kalman (1960). In the so-called Kalman filter (KF), two (sets of) equations are integrated forward in time; one describing the model dynamics and one describing the evolution of the model error covariance. At measurement times, the model prediction and the forecast error covariance are updated due to a minimum variance analysis scheme. A linear process, which is initially described by Gaussian statistics, will remain Gaussian at all times. This makes it possible to derive an exact equation for the evolution of the model error covariance (for more information, see e.g., Jazwinski, 1970 or Bennett, 1992). A nonlinear process, for which the initial condition is taken from a Gaussian distribution, may develop non-Gaussian statistics during nonlinear evolution. In this case, a popular approach has been to use the so-called extended Kalman filter (EKF), where one applies a statistical reduction (approximation) in the equation describing the evolution of the model error covariance to get a closed system of equations. To be more specific, contributions from statistical moments of order three or higher are discarded (e.g., Jazwinski, 1970; Section 3). Various problems regarding the above closure scheme have been reported (e.g., Evensen, 1992). This has lead to extensive studies, and several alternative data assimilation schemes have been developed over the last years, among them the ensemble Kalman filter (EnKF).

The EnKF allows for a consistent integration of observations and the numerical model of a particular (linear or non-linear) dynamical system. An ensemble of model states is used to represent the model solution and its uncertainty. A flowchart showing the EnKF implementation is shown in Figure 2.

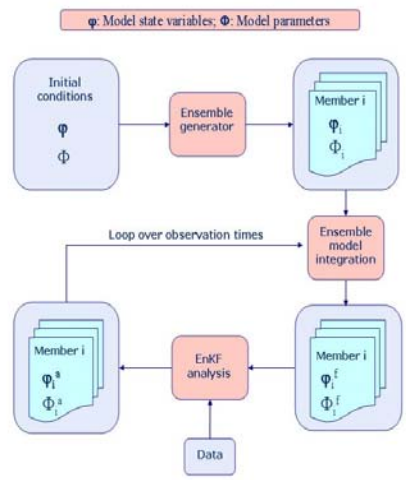

Figure 2. Flowchart showing the implementation of the EnKF. Let $\boldsymbol{\psi} = (\varphi, \boldsymbol{\varphi})$ be the combined state and parameter vector. That is, let φ be a vector containing the model state at a given time (e.g., oil production) and $\boldsymbol{\varphi}$ a vector containing all the model parameters (e.g., porosity). The EnKF then uses an ensemble of such states which is initially generated by a statistical simulator. Each member of the ensemble is then integrated forward in time by the reservoir simulator until a time where data are available, giving a forecast φ^f with given parameters $\boldsymbol{\varphi}^f$. At data times, the ensemble is updated based on information from the data and the underlying error statistics, giving an updated ensemble which is conditioned on the data. The ensemble integration and EnKF analysis process is then repeated in a loop over all observation times.

In state estimation problems the state vector holds the dynamical model state variables as defined on the model grid. However, in combined state and parameter estimation the poorly known model parameters can also be included in the state vector. Thus, let $\boldsymbol{\Psi}$ be a vector containing the discrete model state at a particular time as well as poorly known model parameters.

In the EnKF an ensemble of realizations is used to represent the true error statistics, i.e., let $\boldsymbol{\psi}_j$ denote member j in the ensemble. The ensemble mean $\overline{\boldsymbol{\psi}}$ is usually chosen as the best estimate, while the solution uncertainty is represented by the ensemble covariance matrix, $\mathbf{P}_e$, around the ensemble mean;

$\mathbf{P} \approx \mathbf{P}_e = \overline{(\boldsymbol{\psi} - \overline{\boldsymbol{\psi}})(\boldsymbol{\psi} - \overline{\boldsymbol{\psi}})^T}$, where the overlines denote an average over the ensemble. Any higher order statistical moment can also be estimated from the ensemble, e.g., to study if the ensemble becomes non-Gaussian.

The experiments presented in this paper rely on an ensemble of 100 members. This should be sufficient to provide reliable estimates of the error statistics (e.g. Natvik and Evensen 2003a).

Initialization

Within the EnKF methodology an initial ensemble of model states needs to be generated (Figure 2). This is done by some kind of perturbation of the original initial state, and the resulting ensemble should resemble the true uncertainty. In principle, all model variables can be perturbed, but it can be difficult to specify the correct covariances between model variables in a multivariate model state. Thus, normally one perturbs one or a few of the dominant variables and then spinning up the ensemble during a short integration to develop the correct multivariate statistics. The dominant errors of the model are expected to be associated with the specification of the permeability and porosity, which are perturbed to create the initial ensemble.

Statistical information from geological data is used to generate the initial ensemble. In our case, information about correlation lengths and orientation, vertical layer correlation and correlation between porosity and log-permeability are given.

Ensemble integration

A prediction of the error statistics is generated by integrating the ensemble of model states forward in time, using the deterministic numerical model with an additional stochastic forcing which represents model errors (Figure 2). The general evolution of the model state can be written as $d\boldsymbol{\psi} = \mathbf{f}(\boldsymbol{\psi})dt + \mathbf{g}(\boldsymbol{\psi})d\mathbf{q}$. This equation implies that a model state increment $d\boldsymbol{\psi}$ can be described by the forward model over the time increment considered, $\mathbf{f}(\boldsymbol{\psi})dt$, plus an additional term $\mathbf{g}(\boldsymbol{\psi})d\mathbf{q}$, called the stochastic forcing and representing (unknown) errors in the model over the same time interval.

During integration, the ensemble variance is likely to increase due to unstable model dynamics and the stochastic representation of the model errors. If the model parameters contain errors, the ensemble mean will also drift away from the truth represented by the data (Figure 3).

Data assimilation

The ensemble is updated at times where measurements are available, using the EnKF variance minimizing analysis scheme (Figure 2). It is essential that the observations are treated as random variables with a mean distribution equal to the original data and an error covariance equal to $\mathbf{R}_e$. Thus, an ensemble of observations can be written as $\mathbf{d}_j = \mathbf{d} + \boldsymbol{\varepsilon}_j$, where $\boldsymbol{\varepsilon}_j$ represents the observation errors and $\mathbf{R}_e = \overline{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T}$.

The analysis step for the EnKF is performed on each of the ensemble members and can be described by $\boldsymbol{\psi}_j^a = \boldsymbol{\psi}_j^f + \mathbf{P}_e^f \mathbf{H}^T (\mathbf{H}\mathbf{P}_e^f \mathbf{H}^T + \mathbf{R}_e)^{-1} (\mathbf{d}_j - \mathbf{H}\boldsymbol{\psi}_j^f)$, where superscripts a and f denote the analysis and forecast, respectively. Thus, $\boldsymbol{\psi}_j^f$ represents the state after a forward simulation of ensemble member j to a time when the data assimilation is to be performed, while $\boldsymbol{\psi}_j^a$ is the corresponding analyzed state after the assimilation of the data. Further, $\mathbf{H}$ is the measurement operator relating the data to variables contained in the model state. If the mean is considered to be the best estimate, it is arbitrary whether one updates the mean, $\overline{\boldsymbol{\psi}}$ using the original observations or whether one updates each member, $\boldsymbol{\psi}_j$, based on the perturbed observations. However, the second approach must be implemented in order to obtain the correct error statistics for the analysis (see Evensen 2003 for details).

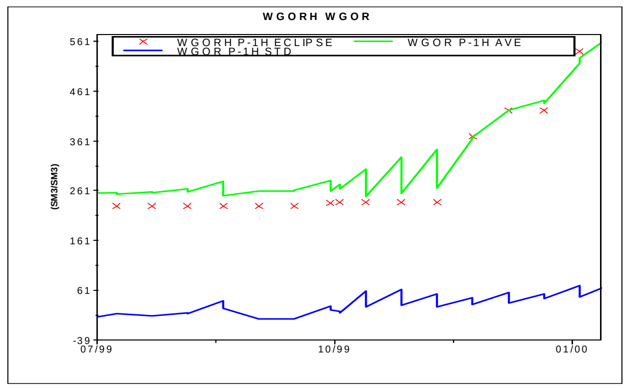

Figure 3. The GOR development for one of the producers is used to illustrate the sequential nature of the EnKF method. Crosses indicate the observations, while the green and blue staircases represent the ensemble mean and the ensemble standard deviation (STD), respectively. Between two data points, the ensemble mean GOR tends to drift away from the data and the STD (ensemble spread) tends to increase. At each EnKF analysis the GOR is brought closer to the observations the STD decreases.

During the EnKF analysis, the realizations will become closer to the observations and thus decrease the ensemble spread. The example in Figure 3 is taken from the GOR development from one of the producers and will be described further in the results section.

In this setup of the EnKF, the dynamic variables pressure, gas and water saturations, Rs and Rv are updated, as well as the static parameters permeability and porosity. The model predicted well variables THP, BHP, OPR, GPR, WPR, WCT and GOR are needed to update the model state at analysis times. The permeability and porosity are indirectly correlated with the well observations through the model state variables.

The measurement errors are drawn from a Gaussian distribution with mean zero and the following standard deviations:

- Bottom hole pressure (WBHPH): 10 %,
- Oil production rate (WOPRH): 15 %,
- Water cut (WWCT): 20 %
- Gas-oil ratio (WGORH): 15 %.

Any data set may contain problematic data which could make the ensemble corrupt, leading to model instabilities. Thus, it is desirable to implement some kind of filter to ensure that such outliers are properly handled. For example, a criterion is used where the distance between the model and the data is compared with the sum of the predicted and measurement standard deviation. If the model and measurements are too different observation errors are increased such that the influence of the data on the analysis decreases.

Results

The model has been run for a 5 year period, from January 1999 to January 2004.

The impact of the assimilation will be discussed through comparison with a reference simulation and also with a prediction simulation, where the updated porosity and permeability fields have been used. The following is an explanation of the different cases discussed throughout the paper.

The "Refcase" represents the eclipse simulation where a traditional history matching approach was used; i.e. tuning parameters manually. The simulation represents the production data quite well, except for deviations in the GOR and water cut development for specific time steps. For example in Figure 4 it is clearly seen that the GOR is overestimated in the Refcase simulation.

The "Assimilation" case represents the ensemble Kalman filter simulation. Observations from four wells were used in the EnKF updates. The standard deviation (STD) shows the spread of the ensemble representing errors in the simulation.

The final porosity and permeability fields after 5 years of assimilation were used to re-initialise/rerun the Refcase simulation ("Updated Refcase"). The reason for this is to study the effect the improved permeability and porosity fields have on a single model integration.

An ensemble integration without assimilation was also studied. The difference from the Refcase was insignificant, thus implying the need for utilizing information in the observations.

Effect on production data

Observed data from the four producers were assimilated typically twice a month using EnKF, with the aim to better represent the production data than in the Refcase.

Figures 4 -7 show the results for the four producers, P-1, P-2, P-3 and P-4. In these plots the oil (WOPR) and water rates (WWPR) as well as the gas oil ratio (GOR) have been plotted. In each of the figures the observations (red crosses), the Refcase (black line), the assimilation (green) and the standard deviation (blue) are shown.

For all four producers the assimilation experiment manages to reproduce the observed data much better than the Refcase. For example in Figure 4 (top) showing producer P-1, the assimilation is capable of capturing the gas breakthrough in April 2000, while it is clearly seen that the Refcase overestimates GOR considerably for this period and also for the following 1 ½ years. This is a clear effect of the EnKF being a sequential method, thus using information from production data whenever available. Smart well applications will in such a matter be very useful since continuous production information is gained.

Figure 4 (middle) shows the oil rate. Here the effect of better representing the gas breakthrough is also seen since oil rate is better reproduced in the same period. This is important for reserve estimation.

The water production (Figure 4, bottom) is also better captured, especially for the first 2 years. There are however a few deviations, especially around May 2000 where the data shows a low water production. Overall the assimilation experiment does a better job than the Refcase.

The observations are perturbed using a specified standard deviation (STD) which will influence the results (as described above). Thus, by changing the observation STD in the water data, the water rates might be better represented (see sensitivity section). The development of the STD is shown in blue for all the parameters and provides an uncertainty estimate of the mean of the ensemble; i.e. reflecting the ensemble spread. In Figure 4 (bottom) the STD is increasing during the first year due to dynamic instability. In this period the changes are large and the ensemble spread will also be large. However, the EnKF is able to decrease the spread shortly after the first peak in the STD. The realizations will become closer to the observations every time new data are assimilated and thus decrease the ensemble spread (see also Figure 3). Likewise the spread will tend to increase during the forward model integration of the ensemble.

Furthermore, for the producer P-2 (Figure 5, top) the EnKF manages to reproduce the gas breakthrough contrary to the Refcase. Thus a better representation of the oil production is gained. The assimilation also better reproduces the water production, although not sufficiently. The largest STD error has been assigned the water production data, which could be improved by more accurate data.

The results from the last two producers P-3 and P-4 as seen in Figure 6 and Figure 7 are similar. Again the EnKF manages to reproduce the gas breakthrough (top) contrary to the Refcase, thus a better representation of the oil production is achieved. The water production is also better matched.

The mismatch in the simulated profiles with the historical water production rates for each of the producers can possibly be explained by the uncertainty in water rate measurements for these low water rates (50-100 Sm3/d) during testing. However, on a template level the measurements under testing are more accurate. Segregation in the pipelines may also generate an additional uncertainty in the measurements.

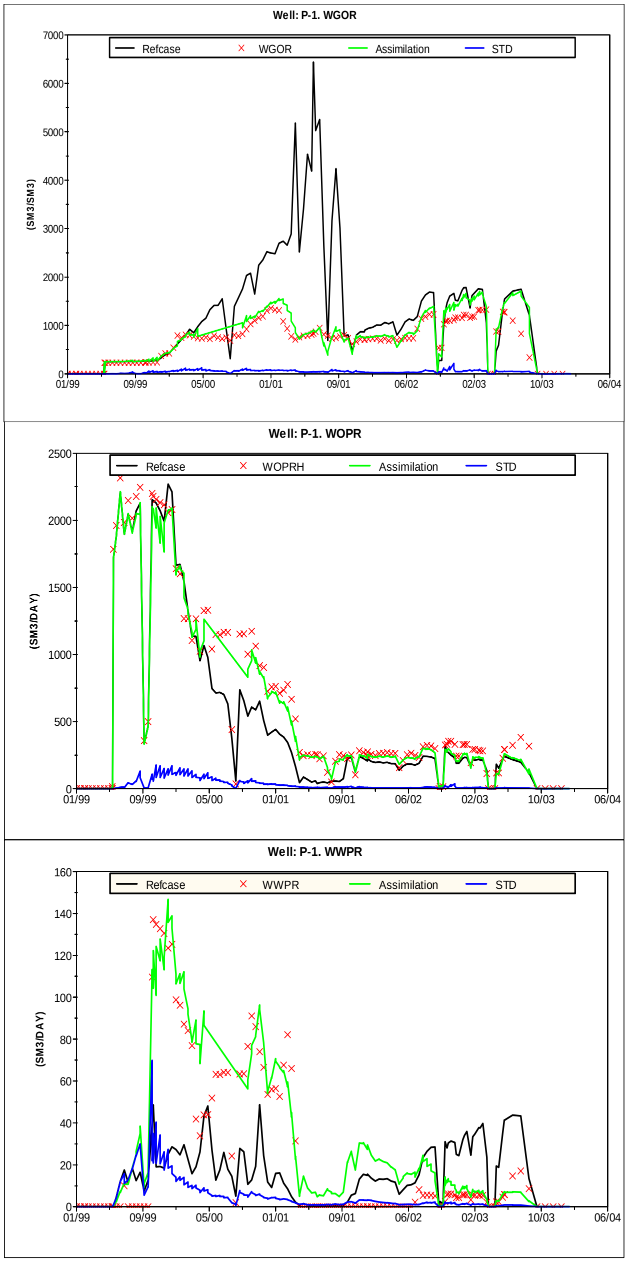

Figure 4. Well: P-1. Refcase and assimilation experiment for WOPR, WGOR (top) WGPR and WWPR (bottom).

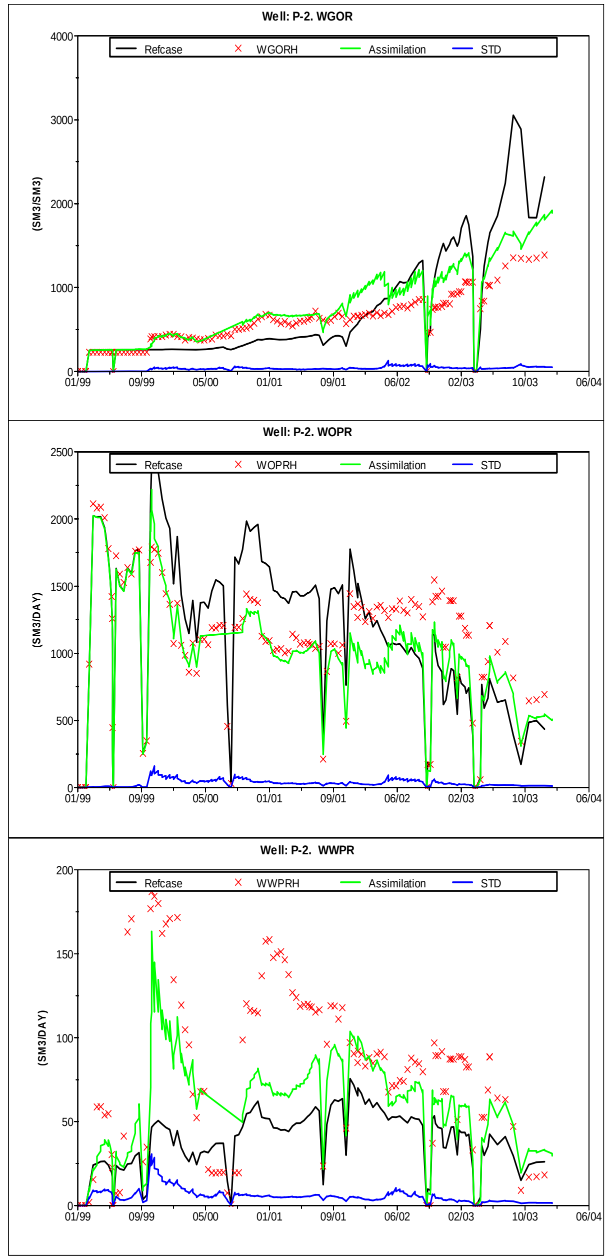

Figure 5. Well: P-2. Refcase and assimilation experiment for WGOR (top), WOPR (middle) and WWPR (bottom).

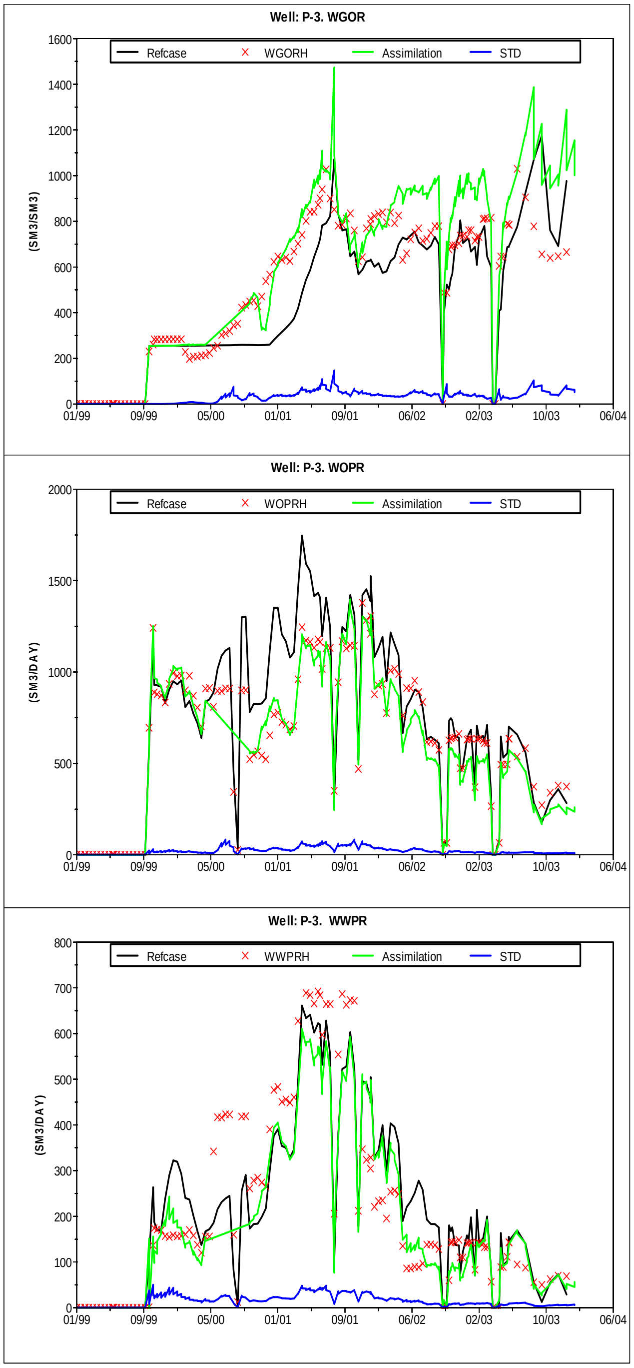

Figure 6. Well: P-3. Refcase and assimilation experiment for WGOR (top), WOPR (middle) and WWPR (bottom).

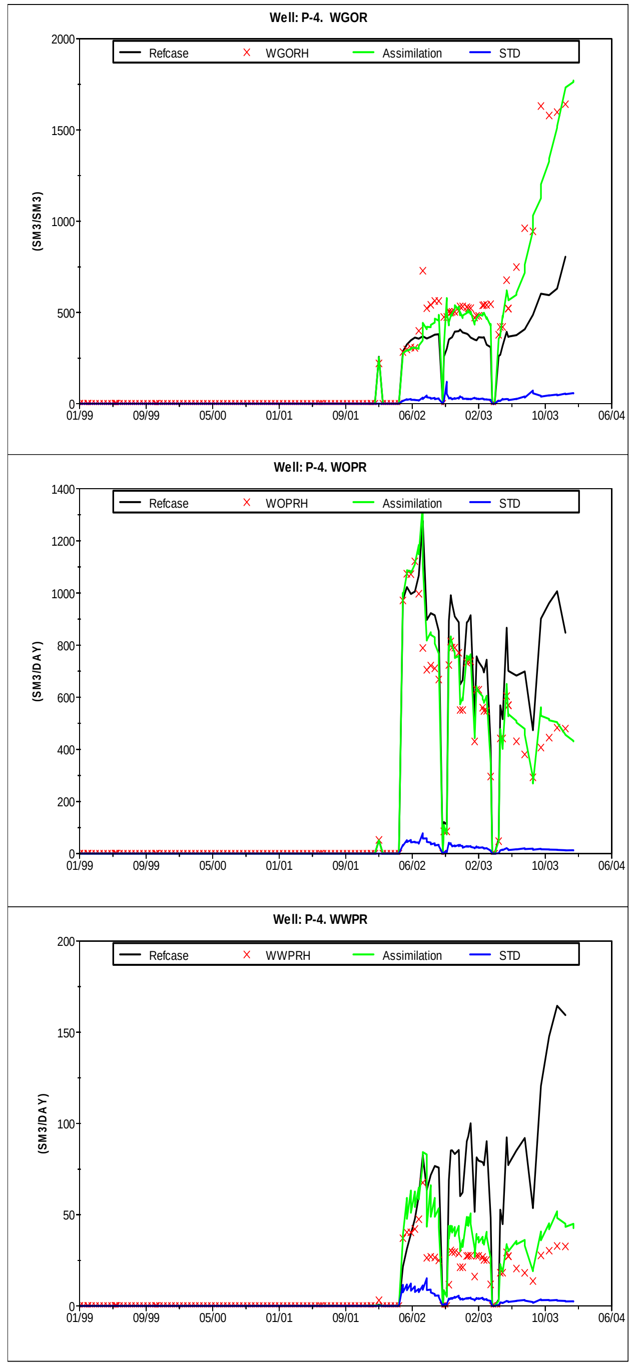

Figure 7. Well: P-4. Refcase and assimilation experiment for WGOR (top), WOPR (middle) and WWPR (bottom).

Effect on porosity and permeability

Normally the porosity and permeability fields are considered as static in a simulation. However, they are in this experiment allowed to contain errors and are updated during the sequential model integration and data assimilation, typically twice a month.

Figure 8 shows the difference in the initial fields and the updated fields after 5 years of assimilation.

The assimilated results might over- or underestimate the “true” porosity and permeability to compensate for other variables which are not updated, e.g. lack of transmissibility across faults. Generally one has the possibility to update other variables; like MULTZ, OWC, GOC, transmissibility across faults and so on.

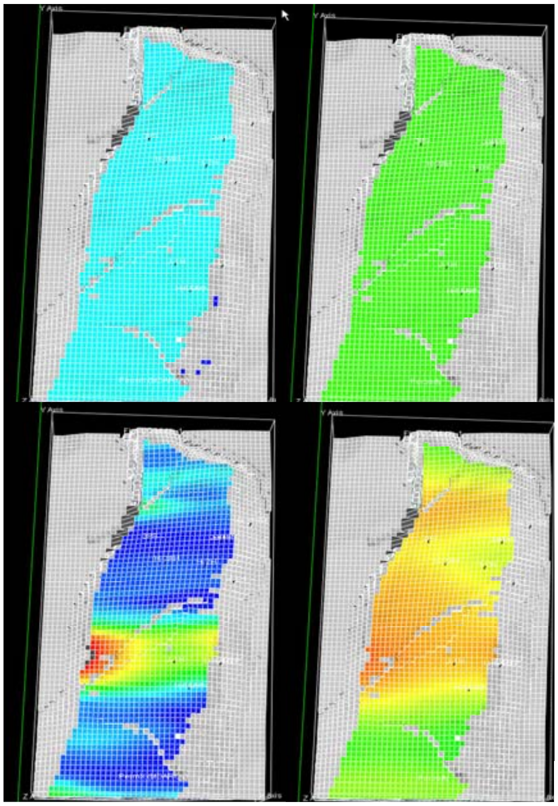

Figure 8. Permeability (left) and porosity (right) fields. Initial condition (top), and after 5 years of assimilation (bottom). Note that fixed axis scale makes the initial fields look very smooth.

Sensitivities

In addition to the base case, two sensitivity experiments were run, one low and one high uncertainty case with respect to measurement errors (Table 1). One assimilation experiment with high uncertainties, but without the measurement filter was also run. It is in general difficult to decide how much one should trust the measurements. As already discussed, water production measurements are known to be associated with high uncertainty in this field case.

Table 1: Sensitivity experiments with respect to measurement errors.

	Low uncertainty case (STD)	Base case (STD)	High uncertainty case (STD)
WBHP	5 %	10 %	20 %
WOPR	10 %	15 %	25 %
WWCT	15 %	20 %	30 %
WGOR	10 %	15 %	25 %

Figure 9 shows the comparison of the cases from Table 1 for the water-cut for each of the producers. The water production measurements are strongly influenced by the measurement filter, especially for the wells P-1 and P-2. During the first two years of production, about half of the measurements for well P-2 are affected by the filter. The same is seen for well P-1 during the last two years of production. This affects the solution which is further away from the measurements. Apart from these two periods, the results are fairly insensitive to changes in the uncertainties. For the oil production rate and gas-oil ratio, the results seem to be insensitive to the size of the measurement error (not shown).

The water-cut results from the assimilation where no filter was applied, is closer to the measurements than the other simulations. This is expected since the results will be more strongly affected by the data without a measurement filter. This should however, not be used as an argument for not applying the filter, but rather serve as an example where probably a too weak criterion was used. In fact we experienced the drawback of not using the filter during our first attempts, where instabilities occurred in some of the simulations.

The experiments imply that the choice of observation uncertainty seems to be of less importance. The assimilation gives an improved estimate in all the cases presented here compared to the Refcase.

Comparison with traditional history matching

The updated porosity and permeability fields (Figure 8, bottom) after 5 years of assimilation were used to rerun the model. These fields are now considered static throughout the simulation period. This was done to see if these new fields alone give a better representation of the fields' characteristics.

Figure 10 shows the GOR development for the Refcase, assimilation and updated Refcase experiments. Generally all peak intervals are better represented using the updated fields than the traditional history matched model (Refcase). Overall the updated Refcase experiment does a good job. However, it is clear that the assimilation experiment shows the best match. This is expected since in the EnKF assimilation allows the porosity and permeability fields to enter different regimes.

For example for well P-1 (Figure 10 top), there is a better match during the beginning and at the end of the simulation period than from April 2001 to July 2002. This might indicate that other time dependent variables should have been included in the assimilation experiment. This will be implemented in the next phase and is an ongoing activity in the project. The other variables, oil and gas production and bottom hole pressure were better represented using the updated fields than the Refcase simulation, although not as good as the assimilation experiment (not shown). There are some deviations from the water production (not shown) although it better captures the true field represented by the observations than the Refcase experiment.

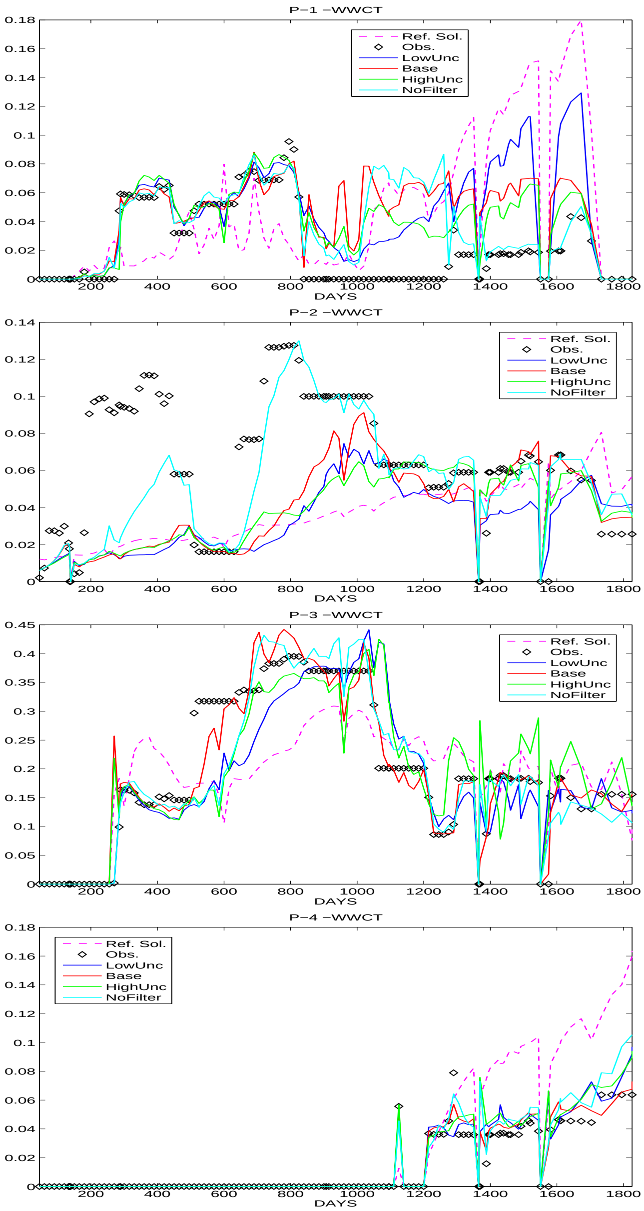

Figure 9: Comparison of water-cut results when assimilations are run with uncertainties shown in Table 1. Well P-1, P-2, P-3, and P-4 are shown from top to bottom.

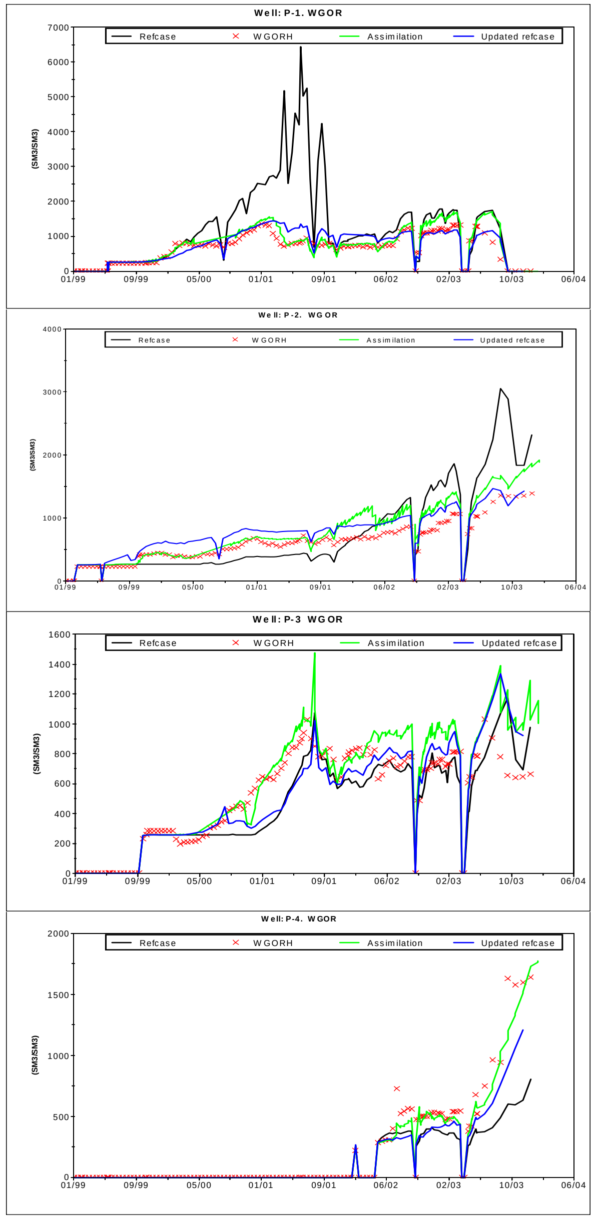

Figure 10. New reference run using updated porosity and permeability fields for showing the WGOR for all four oil producers.

Conclusion

The possibility of improving model parameters such as permeability and porosity by assimilating data such as bottom hole pressure and production rates of oil, gas and water will be beneficial for further planning of field developments. It has been demonstrated that by using the EnKF in a field implementation, a better history matched model could be achieved with improved porosity and permeability estimates. The updated fields were also used to rerun the model, giving a better match to the production data than the original history matched simulation. Further, since the model state is also updated with measurements, a good estimate of the current state of the reservoir is obtained at the end of the model simulation, which forms the optimal starting point for computing predictions. Thus, the methodology forms an ideal framework for operational reservoir monitoring and prediction.

The experiment shows the necessity to allow for the updating of other variables in the assimilation using EnKF, such as transmissibility across faults, MULTZ and the possibility to update the gas-oil contact and/or oil-water contact.

Acknowledgement

The authors would like to thank Tina Todnem for providing the geostatistics and also Hydro, Petoro and Statoil for permission to publish this paper. The work done by Berg, Flornes and Nævdal has been done with financial support from the ROAW project, financed by Norwegian Research Council and industrial partners.

References

Bennett, A.F., 1992. Inverse Methods in Physical Oceanography Cambridge Univ. Press, Cambridge, ISBN 0-521-38568-7.

Evensen, G., 1992. Using the extended Kalman filter with a multilayer quasi-geostrophic ocean model. J. Geophys. Res. 97 (C11), 17905–17924.

Evensen, G., Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics, J. Geophys. Res., 99(C5), 10,143-10,162, 1994a.

Evensen, G., The ensemble Kalman filter: Theoretical formulation and practical implementation, Ocean Dynamics, 53 , 343-367, 2003.

Gao, G., and A. C. Reynolds, Quantifying the uncertainty for the PUNQ-S3 problem in a Bayesian setting with the RML and EnKF, SPE reservoir simulation symposium (SPE 93324), 2005.

Gu, Y., and D. S. Oliver, History matching of the PUNQ-S3 reservoir model using the ensemble Kalman lter, SPE Annual Technical Conference and Exhibition (SPE 89942), 2004.

Haugen VE, Evensen G (2002). Assimilation of SST and SLA data into an OGCM for the Indian Ocean. Ocean Dynamics 52: 133-151.

Houtekamer, P. L., H. L. Mitchell, G. Pellerin, M. Buehner, M. Charron, L. Spacek, and B. Hansen, Atmospheric data assimilation with an ensemble Kalman filter: Results with real observations, Mon. Weather Rev., 133 , 604-620, 2005.

Jazwinski, A.H., 1970. Stochastic Processes and Filtering Theory. Academic Press, New York.

Kalman, R.E., 1960. A new approach to linear filter and prediction problems. J. Basic Eng. 82, 35– 45.

Lantuejoul, C., Geostatistical Simulation: Models and Algorithms, Springer-Verlag, 2002.

Liu, N., and D. S. Oliver, Critical evaluation of the ensemble Kalman filter on history matching of geologic facies, SPE reservoir simulation symposium (SPE 92867), 2005a.

Liu, N., and D. S. Oliver, Ensemble Kalman filter for automatic history matching of geologic facies, J. Petrolium Sci. and Eng., 47 , 147{161, 2005b.

Lorentzen, R. J., K. K. Fjelde, J. Frøyen, A. C. V. M. Lage, G. Nævdal, and E. H. Vefring: "Underbalanced and Low-head Drilling Operations: Real Time Interpretation of Measured Data and Operational Support", paper SPE 71384 presented at the 2001 SPE Annual Technical Conference and Exhibition held in New Orleans, Lousiana, 30 September – 3 October 2001.

Lorentzen, R. J., G. Nævdal and A. C. V. M. Lage: "Tuning of parameters in a two-phase flow model using ensemble Kalman filter." Int. J. Multiphase Flow, Vol. 29, No. 8, pp. 1283-1309, 2003.

Lorentzen, R. J., G. Nævdal, B. Valles, A. M. Berg, and A.-A. Grimstad, Analysis of the ensemble Kalman filter for estimation of permeability and porosity in reservoir models, SPE 96375, 2005.

Natvik L-J, Evensen G (2003a) Assimilation of ocean colour data into a biochemical model of the North Atlantic. Part 1. Data assimilation experiments. JMS, 40-41, 127-153.

Natvik L-J, Evensen G (2003b) Assimilation of ocean colour data into a biochemical model of the North Atlantic. Part 2. Statistical analysis. JMS, 40-41, 155-169.

Nævdal, G., T. Mannseth, and E. Vefring, Near well reservoir monitoring through ensemble Kalman filter, Proceeding of SPE/DOE Improved Oil recovery Symposium (SPE 84372), 2002.

Nævdal, G., L. M. Johnsen, S. I. Aanonsen, and E. Vefring, Reservoir monitoring and continuous model updating using the ensemble Kalman filter, SPE Annual Technical Conference and Exhibition (SPE 84372), 2003.

Skjervheim, J.-A., G. Evensen, S. I. Aanonsen, B. O. Ruud, and T. A. Johansen, In corporating 4D seismic data in reservoir simulatuion models using ensemble Kalman filter, SPE 95789, 2005.

Wen, X.-H., and W. H. Chen, Real time reservoir model updating using the ensemble Kalman filter, SPE reservoir simulation symposium (SPE 92991), 2005.