Reply

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1. Introduction

In his comment Dr. van Leeuwen (1999) discusses two main points. First, he gives a derivation of the ''inbreeding'' and ''suboptimality'' terms in the ensemble Kalman filter (EnKF) of Evensen (1994) and in the double ensemble Kalman filter (DEnKF), which we proposed in our paper (Houtekamer and Mitchell 1998). Second, he expresses his concern about the use of small ensembles.

While we thank Dr. van Leeuwen for the clarifications, we would like to briefly summarize his conclusions, add some comments of our own, and compare his conclusions with our earlier findings.

2. Symmetry of the DEnKF

In his Eq. (15), Dr. van Leeuwen defines $\mathbf{P}_1$ and $\mathbf{P}_2$ to be the true error covariances of the two ensembles in the DEnKF. It may be noted here that, due to the symmetric design of the DEnKF, the true covariances $\mathbf{P}_1$ and $\mathbf{P}_2$ will be identical at all times. Exploiting this identity would simplify Eq. (20).

Also, from the remark below Eq. (16) it appears that $\mathbf{K}_2$ contains first- and second-order terms in ϵ_2. Therefore, the ''zeroth''- and ''first''-order terms in Eq. (20) may contain first-, second-, and higher-order terms in ϵ_1 and ϵ_2. A similar comment applies to the symbol δ. Thus, the use of $\mathbf{K}_2$ (rather than $\mathbf{K}$) and δ makes it difficult to interpret Eq. (20) as a series expansion.

3. Inbreeding and suboptimality terms

In his Eq. (19), Dr. van Leeuwen gives the inbreeding and suboptimality terms for the EnKF. The first of these terms arises as the product of first-order terms in the forecast-error covariance and the gain, while the second term is due to second-order terms in the gain. For the EnKF the inbreeding term is negative and leads to ensemble variances that are smaller than the optimal variances that would be computed using a standard Kalman filter. This clearly indicates that there is a problem with the EnKF approach, since it is impossible for the analysis error variances to be smaller than the optimal values. The (smaller) suboptimality term quantifies the inevitable degradation (i.e., suboptimality) with respect to a standard Kalman filter due to a finite ensemble size. In summary, we note that for the EnKF it is the inbreeding term that causes the ensemble covariances to be unrepresentative of the ensemble mean error. In extreme cases, with very small ensembles, this may cause failure of the EnKF as noted by van Leeuwen and confirmed experimentally in the upper-left-hand panel of our Fig. 3.

For the DEnKF, at the first analysis step, the inbreeding term [see Eq. (20) of Dr. van Leeuwen] has zero expectation value, because it contains products of ϵ_1 and ϵ_2. In subsequent assimilation steps, some inbreeding will occur but it is not clear from either our paper or the comment by van Leeuwen in which sense it will act. As before, the suboptimality term, which is positive definite, quantifies the inevitable degradation with respect to a standard Kalman filter due to the use of an ensemble of finite size. The degradation due to a finite ensemble size is also clearly visible in our Fig. 5, for instance.

4. The use of small ensembles

In practice the maximum ensemble size will be determined by what is computationally feasible. A minimal ensemble size follows from the requirement that the ensemble correlations be more accurate than those obtained with alternative methods, such as statistical interpolation, 3DVAR, or 4DVAR, being used operationally at present. It is not at all clear that the right standard of comparison should be the 5% error that Dr. van Leeuwen uses in his concluding discussion. Perhaps even allowing inaccuracies of 10%–20% in a DEnKF would result in more accurate analyses than those being produced by currently operational schemes.

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5. Conclusions

We agree with Dr. van Leeuwen that more research is needed. Other factors such as model error and effects of nonlinearity may also have an impact on the ensemble size and configuration that one will decide to use. It may not be very useful to employ extremely large ensembles if actual analysis accuracy is mostly limited by other shortcomings of the data assimilation system (such as a systematic error in the forecast model used to generate the ensemble of first-guess fields). In practice, there will also be trade-offs between ensemble size and resolution, for example, it may be interesting to consider reducing ensemble size to permit increased spatial or temporal resolution.

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